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Brane/flux annihilation transitions and nonperturbative moduli stabilization

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ABSTRACT: By extending the calculation of Kähler moduli stabilization to account for an embiggened antibrane, we reevaluate brane/flux annihilation in a warped throat with one stabilized Kähler modulus. We find that depending on the relative size of various fluxes three things can occur: the decay process proceeds unhindered, the $\overline{D3}$ -branes are forbidden to decay classically, or the entire space decompactifies. Additionally, we show that the Kähler modulus receives a contribution from the collective 3-brane tension. This allows for a significant change in compactified volume during the transition and possibly mitigates some fine tuning otherwise required to achieve large volume.

KEYWORDS: Flux compactifications, D-branes, Superstring Vacua

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1 Introduction

The annihilation of antibranes in the absence of brane partners occurs in a process known as brane/flux annihilation [1]. It is well known that if a collection of D-branes form a stack, the matrix of their positions becomes non-abelian. In the presence of other fluxes, the famous Myers effect [2] occurs, causing the position matrix to become an irreducible representation of $SU(2)$ and effectively generating two new directions for the brane. Thus a stack of $\overline{D3}$ -branes “embiggens”¹ to become a single 5-brane. The 5-brane can unwrap

¹This term was applied to the expanded brane in [32]. We prefer the descriptive term “embiggened brane” to the more enigmatic “giant graviton” which also appears in the literature.

itself by consuming one unit of H_3 flux while traversing a 3-cycle, in the process decaying into a new collection of D3-branes. This process has been studied in the context of the Klebanov-Strassler solution for a warped deformed conifold throat [3], with the expectation that such a throat would be part of a larger compact geometry [4, 5]. Understanding these transitions is essential for building a knowledge of the landscape of string vacua, in particular the history of a universe transitioning within it and the stability of the various solutions; see for example [6]–[14].

Realistic string vacua must have stabilized moduli, and as is well-known, in type IIB compactifications fluxes generically only stabilize the complex structure moduli and dilaton. The most well-studied mechanism for Kähler moduli stabilization is the introduction of nonperturbative physics hosted on a 4-cycle, either wrapped 7-branes with strong coupling dynamics or instantonic Euclidean 3-branes, as put forward by KKLT [6]. Such a mechanism can stabilize Kähler moduli, but in general introduces a potential for 3-branes as well. After being described in [15–17], the detailed form of this potential was worked out by Baumann et al. [18], and both supersymmetric [19] and nonsupersymmetric [20] vacua for the combined system of Kähler and brane moduli were worked out; for other work using this potential, see [21]–[30]. Moreover, it was argued in [19, 20] that despite their lack of supersymmetry, $\overline{D3}$ -branes feel a similar potential in the presence of the nonperturbative physics.

The question thus naturally arises how the nonperturbative moduli stabilization modifies the brane/flux annihilation decay, and therefore how the stability of the landscape of string vacua is affected by nonperturbative moduli stabilization. It is this question that we study in this paper. We argue that as the nonperturbative physics is determined to leading order by the backreaction of the brane tension on the warped volume associated to the 4-cycle [18], the effective tension of the embiggened brane is the relevant quantity. Calculating the total potential coming from both brane/flux and nonperturbative effects in the case of a single Kähler modulus, we show that depending on the parameters of the solution (in particular the overall size of the geometry and the depth of the warped throat, as well as the value of the flux superpotential), a number of different results can occur.

It is possible for the brane/flux potential to dominate, and for the decay process to go through unchanged as described by [1]. Other possibilities exist as well, however. The nonperturbative potential is negative, and if it dominates, the decay is in general shut down and the world left with negative cosmological constant. An interplay between the two potentials can exist such that a vacuum that was unstable without the nonperturbative effects can be stabilized near zero vacuum energy. And finally, vacua with positive cosmological constant may find that they cannot complete the decay, as they experience a classical instability to spontaneous decompactification to ten dimensions during the brane/flux annihilation process. This can occur even for antibranes that are stable against decay to large volume before the Myers effect takes place.

This paper is organized as follows. In §2 we establish the setting by reviewing the physics responsible for brane/flux annihilation in the warped throat. In §3 we investigate the non-perturbative physics underlying moduli stabilization and argue for the effect of an embiggened, decaying stack of antibranes on the dynamics. We also choose a particular embedding for the nonperturbative 7-brane, the “simplest Kuperstein” embedding [31],

which respects the symmetries of the decay process. In §4 we discuss the requirements for volume stabilization, and the beginning- and end-points of the annihilation process. We then consider the possible potentials along the path of the decay in §5, and give a summary and example in §6. We conclude in §7.

2 Review of brane/flux annihilation

2.1 Warped throat

The arena for our investigations is a warped throat of Klebanov-Strassler (KS) type [3], attached to a compact bulk geometry; we expect the results to be similar for more intricate warped geometries [32–34]. In the absence of nonperturbative moduli stabilization and $\overline{D3}$ -branes, the background is of imaginary self-dual type [5]. The metric takes the form²

$$ds^2 = e^{2A(y)} e^{-6u(x)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} e^{-2A(y)} \tilde{g}_{mn} dy^m dy^n, \quad (2.1)$$

where e^A is the warp factor and e^{2u} is the Weyl rescaling required to decouple the overall volume mode from the four-dimensional graviton; we shall always assume e^{2u} as constant over the noncompact space.³ The five-form takes the form

$$\tilde{F}_5 \equiv dC_4 - C_2 \wedge H_3 = (1 + *) \frac{1}{g_s} [d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3], \quad (2.2)$$

where for imaginary self-dual backgrounds [17],

$$\alpha(y) = e^{-12u} e^{4A}(y). \quad (2.3)$$

Three-form fluxes F_3 and H_3 are also generically present in the geometry, obeying the imaginary self-dual condition $*_6 G_3 = iG_3$ with $G_3 \equiv F_3 - \tau H_3$; we will assume for simplicity constant dilaton $e^\Phi = g_s$ and vanishing RR zero-form potential $C_0 = 0$, so $\tau \equiv C_0 + ie^{-\Phi} \rightarrow i/g_s$.

We will be focused on the dynamics inside a Klebanov-Strassler throat associated with a particular deformed conifold singularity with 3-cycles A, B . The throat is supported by crossed fluxes wrapped on the 3-cycles,

$$\frac{1}{4\pi^2} \int_A F_3 = M, \quad \frac{1}{4\pi^2} \int_B H_3 = -K, \quad (2.4)$$

where we have taken $\alpha' = 1$. These fluxes stabilize the complex structure modulus ϵ at an exponentially suppressed value [5],

$$\epsilon \approx e^{-\pi K/g_s M}, \quad (2.5)$$

²We use the conventions of [38], which differ from [5] by an explicit factor of the string coupling in the definition of the Einstein metric in terms of the string metric, $g_{MN}^E = g_s e^{-\Phi} g_{MN}^{\text{str}}$; in these conventions the string and Einstein metrics coincide for the case of constant dilaton.

³An alternate approach to the realization of the Kähler modulus in the ten-dimensional metric has been studied in [17, 35–37]. The difference does not affect the effective four-dimensional physics we will study.

where ϵ characterizes the deformation of the conifold through its defining equation [39],

$$\sum_{a=1}^4 z_a^2 = \epsilon^2, \tag{2.6}$$

on four complex variables z_a . In general the deformed conifold's metric can be written in terms of a radial variable, a two-sphere, and a three-sphere; the region we are interested in is the tip of the throat, where the two-sphere shrinks to zero size while the three-sphere stays finite. The unwarped metric at the tip is [40],⁴

$$\tilde{g}_{mn} dy^m dy^n = \left(\frac{2}{3}\right)^{1/3} \epsilon^{4/3} \left(\frac{1}{2} d\bar{r}^2 + d\Omega_3^2\right). \tag{2.7}$$

The warp factor, meanwhile, takes the form near the base of the throat [5, 17, 40, 41],

$$e^{-2A} \approx 2^{1/3} g_s M I_0^{1/2} \epsilon^{-4/3} e^{-2u}, \tag{2.8}$$

up to terms of order $\mathcal{O}(\bar{r}^2)$, where $I_0 \approx 0.718$ is a pure number. The total metric at the tip of the throat thus takes the form

$$ds^2 \approx e^{-4u} a_0^2 g_{\mu\nu} dx^\mu dx^\nu + g_s M b_0^2 \left(\frac{1}{2} d\bar{r}^2 + d\psi^2 + \sin^2 \psi d\Omega_2^2\right), \tag{2.9}$$

where we have for future convenience written the three-sphere metric as a coordinate ψ along with the metric for a two-sphere, and defined the constants

$$a_0^2 = \frac{\epsilon^{4/3}}{g_s M (3/2)^{1/3} b_0^2}, \quad b_0^2 = \left(\frac{4}{3}\right)^{1/3} I_0^{1/2} \approx 0.933. \tag{2.10}$$

We note that at the tip of the throat, the compact dimensions are independent of both ϵ and e^{2u} ; both cancel precisely between the unwarped metric and the warp factor. Thus the tip of the throat is independent of the Kahler modulus, with its size set purely by $g_s M$. We shall always take $g_s M \gg 1$ so that the supergravity limit holds at the tip of the throat, as well as $g_s N \equiv g_s M K > (g_s M)^2$ so that $\epsilon \ll 1$.

We assume that this warped throat is part of a larger, compact geometry, though we will make no attempt to characterize the remaining space. Compactness introduces several issues, the first of which is the need for the C_4 -tadpole to vanish, which requires

$$\frac{\chi(X)}{24} = Q_3 + \frac{1}{2\kappa_{10}^2 T_3} \int H_3 \wedge F_3, \tag{2.11}$$

where Q_3 is the total charge in D3-branes and $\overline{D3}$ -branes and $\chi(X)$ characterizes the contribution from the 7-branes on the underlying Calabi-Yau 4-fold X in the F-theory picture. The total space is also characterized by a number of additional complex structure moduli and Kähler moduli; the complex structure moduli can be stabilized by three-form fluxes on the associated cycles [5, 42].

⁴The radial variable here is $\bar{r} \equiv (3/2)^{1/6} \tau$ in terms of the variable τ of [3, 40].

The Kähler moduli, on the other hand, must be stabilized in another fashion; we shall consider the nonperturbative KKLT stabilization [6] associated to 7-branes hosting strong dynamics (or alternately Euclidean D3-branes) wrapping the corresponding 4-cycles. For simplicity, we will consider just one Kähler modulus, associated with the overall volume e^{4u} ; we shall review the nonperturbative stabilization mechanism shortly.

In principle, the end of the KS throat and the compact geometry beyond provide corrections to the geometry and background fields, even at the bottom of the throat [10, 43, 44]. We shall neglect these various corrections, focusing on the influence of only the nonperturbative moduli stabilization effects on the antibrane decay at the bottom of the throat; the moduli stabilization is a universal effect in such models, while any other effects are model-dependent. In general, the validity of this assumption depends on details of the compact geometry, which we won't specify; but even in cases where additional corrections arise, we may expect the general phenomena we will describe still to obtain qualitatively.

2.2 Life and decay of antibranes

Moduli-stabilized backgrounds of the type just discussed generally have AdS vacua preserving $\mathcal{N} = 1$ supersymmetry; to obtain dS vacua with no supersymmetry, one mechanism is to place some $\overline{D3}$ -branes in a the warped throat [6]. The action for a D3- or $\overline{D3}$ -brane in this throat is

$$S_{3\pm} = -\frac{\mu_3}{g_s} \int d^4x \sqrt{-g_4} \pm \mu_3 \int C_4, \tag{2.12}$$

$$= -\frac{\mu_3}{g_s} \int d^4x \sqrt{-\tilde{g}_4} (e^{-12u} e^{4A} \pm e^{-12u} e^{4A}), \tag{2.13}$$

indicating that while a D3-brane feels no force from the background, the $\overline{D3}$ experiences a potential, which both gives a positive contribution to the vacuum energy and draws the brane to the bottom of the throat, where by minimizing the warp factor the brane seeks to reduce its energy.

The tip of the throat, however, prevents the antibranes from escaping to zero warp factor, and thus they may settle down into a metastable vacuum. A further decay mechanism for the antibranes was worked out by Kachru, Pearson and Verlinde (KPV) [1] and elaborated on by [10], in which the p $\overline{D3}$ -branes decay to $M - p$ ordinary D3-branes while changing the flux K by one unit, and this will be the focus of our investigation.

Once our $\overline{D3}$ -branes reach the tip, the surrounding flux attempts to screen them, lowering the effective charge and raising the effective stress energy, producing a net attractive force and causing the $\overline{D3}$ -branes to clump [10]. There are also non-perturbative effects, which we will discuss in detail shortly, that draw the clumping $\overline{D3}$ -branes to the SUSY minimum as if they were D3-branes [20]. Once the branes are together, a new set of dynamics becomes available to them:⁵ the flux will induce them to puff up or “embiggen” into a 5-brane wrapping an S^2 on the tip via the Myers effect [2]. It is then possible for the antibranes to “unwrap” themselves across the S^3 , decaying into ordinary D3-branes in the process.

⁵In contrast to [9], we assume embiggening only occurs after *all* $\overline{D3}$ branes arrive at the tip of the throat.

Here we repeat the results of [1] as a review as well as to set our conventions. We approximate our puffed up $\overline{\text{D3}}$ -branes as one anti-NS5-brane with the appropriate D3-brane charge dissolved in it. The $\overline{\text{NS5}}$ wraps a two-sphere on the tip, and we let ψ be the direction normal to the brane on the tip. The action for our $\overline{\text{NS5}}$ -brane is [38],

$$S = -\frac{\mu_5}{g_s^2} \int d^6 \xi [-\det(g_{\parallel}) \det(g_{\perp} + 2\pi g_s \mathcal{F})]^{1/2} - \mu_5 \int [B_6 + C_4 \wedge 2\pi \mathcal{F}_2], \quad (2.14)$$

where $2\pi \mathcal{F}_2 \equiv 2\pi F_2 - C_2$, and we have separated the metric based on whether it is parallel or perpendicular to the D3-branes which source the $\overline{\text{NS5}}$ -brane. The effective charge and tension of p antibranes is encoded in⁶

$$\int_{S^2} F_2 = 2\pi p. \quad (2.15)$$

The field strength for F_3 can be realized from the potential

$$C_2 = M \left(\psi - \frac{1}{2} \sin 2\psi \right) \sin \theta d\theta \wedge d\phi, \quad (2.16)$$

where θ and ϕ parameterize the two-sphere, while the six-form potentials may be obtained from [38]

$$d \left(B_6 - \frac{i}{g_s} C_6 \right) = \frac{i e^{4A}}{g_s} (*_6 G_3 - i G_3) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 = 0, \quad (2.17)$$

and thus may be chosen to vanish.⁷

We can now evaluate several relevant integrals,

$$\int_{S^2} d\theta d\phi \sqrt{\det g_{\perp}} = 4\pi b_0^2 g_s M \sin^2 \psi, \quad \int_{S^2} 2\pi \mathcal{F}_2 = 4\pi^2 \left(p - \frac{M\psi}{\pi} + \frac{M}{2\pi} \sin 2\psi \right), \quad (2.18)$$

$$\frac{1}{g_s} \int d^4 x \sqrt{\det g_{\parallel}} = \int C_4 = \frac{a_0^4}{g_s e^{8u}} \int d^4 x \sqrt{\tilde{g}_4}, \quad (2.19)$$

where recall \tilde{g}_4 is the determinant of the noncompact components of the metric stripped of the warp factor and Weyl rescaling; this is the effective 4D metric. So we can put everything together to obtain the effective four-dimensional action

$$S = - \int d^4 x \sqrt{-\tilde{g}_4} \frac{\mathcal{C}_0}{e^{8u}} \left[N_3(\psi) \sqrt{1 - Z^2 e^{4u} \dot{\psi}^2} - Q_3(\psi) \right], \quad (2.20)$$

where we have defined,⁸

$$Q_3(\psi) = -p + M \left(\frac{\psi}{\pi} - \frac{1}{2\pi} \sin 2\psi \right), \quad N_3(\psi) = \sqrt{\frac{M^2 b_0^4}{\pi^2} \sin^4 \psi + (Q_3(\psi))^2}, \quad (2.21)$$

$$\mathcal{C}_0 = \frac{4\pi^2 a_0^4 \mu_5}{g_s} = \frac{\mu_3 a_0^4}{g_s}, \quad Z^2 = \frac{g_s M b_0^2}{a_0^2}. \quad (2.22)$$

⁶Note that the correct D3-brane charge can be realized either with positive F_2 on an anti-NS5, or negative F_2 on an NS5. Only the choice we have made leads to the correct potential for antibrane decay, however; the opposite choice can be seen to lead to a potential that gains energy by adding antibrane charge.

⁷In [1] the WZ contribution to the potential was stated to be obtained from B_6 , but this is zero; the relevant term actually comes from $C_4 \wedge \mathcal{F}_2$.

⁸These functions are related to those in [10] by $N_3(\psi) = M V_2(\psi)/\pi$, $Q_3(\psi) = -M U(\psi)/\pi$.

We have chosen to assume that the only variation is the location of the 5-brane in the ψ -direction with time. For the static case, the effective brane/flux potential is⁹

$$V_{\text{bf}}(\psi) = \frac{\mathcal{C}_0}{e^{8u}}(N_3(\psi) - Q_3(\psi)). \quad (2.23)$$

This can readily be interpreted; N_3 and Q_3 are the effective D3-brane tension and D3-brane charge, in units of a single D3, as the embiggened brane undergoes the transition. As the brane moves from $\psi = 0$ to $\psi = \pi$, the charge $Q_3(\psi)$ varies from $Q_3(\psi = 0) = -p$, corresponding to the initial p $\overline{\text{D3}}$ -branes, to $Q_3(\psi = \pi) = M - p$, the number of ordinary D3's in the final state, thanks to the effect of the coupling to C_2 . Meanwhile $N_3(\psi)$ varies between a tension of p branes to a tension of $M - p$, with an additional contribution of the same order of magnitude in the middle stemming from the $\overline{\text{NS5}}$ -brane bubble's finite extent, encoded in the $\sin^4 \psi$ term; this keeps the tension always positive even while the charge is passing through zero.

The $\overline{\text{NS5}}$ -brane is a source for H_3 , and as it sweeps out the three-sphere, it changes the flux of this field (2.4) from $K \rightarrow K - 1$. As a result of this change coupled with the change from p $\overline{\text{D3}}$ -branes to $M - p$ D3-branes, the tadpole constraint (2.11) is preserved, with the only parts changing being the three-brane charge and the fluxes integrated over the cycles associated to our warped throat:

$$-p + KM \quad \rightarrow \quad (M - p) + (K - 1)M = -p + KM. \quad (2.24)$$

This transition also shrinks the throat slightly as $\epsilon \rightarrow e^{\pi/g_s M} \epsilon$, but since $g_s M \gg 1$ the change is small. Because this decay process involves a removal of both antibranes and flux, it is commonly called *brane/flux annihilation*.

3 Structure of the nonperturbative potential

Nonperturbative physics on a seven-brane or Euclidean D3-brane wrapping a 4-cycle Σ can stabilize the overall volume of a warped compactification; however, this mechanism introduces a potential for D3-branes as well. In this section, we review these effects and argue for an expression capturing the corresponding potential on an embiggened brane undergoing the brane/flux annihilation decay process.

3.1 Nonperturbative effects and the embiggened brane

For an ordinary D3-brane, this physics is determined by a superpotential [18]

$$W_{\text{np}}(Y) = W_0 + A(Y)e^{-a\rho}, \quad (3.1)$$

with

$$A(Y) \equiv A_0 e^{-\zeta(Y)} = A_0 f(Y)^{1/n}, \quad (3.2)$$

⁹The same dependence on the Kähler modulus arises in the formalism of [17, 35–37] in the strongly warped limit. We thank B. Underwood for discussions on this point.

where W_0 and A_0 are constants that depend on the (already stabilized) values of the complex structure moduli, Y are complex coordinates for the D3-brane, a is the numerical constant $2\pi/n$, ρ is a complex modulus containing the overall volume:

$$\rho \equiv \frac{1}{2} \left(e^{4u} + \frac{1}{3} k(Y, \bar{Y}) \right) + ib, \tag{3.3}$$

b is the axion related to C_4 integrated over the corresponding 4-cycle, k is the geometric Kähler potential on the space, n is the number of 7-branes/Euclidean D3-branes, and f is their embedding function; that is, the branes sourcing the nonperturbative physics are located at $f = 0$. (Note however that f in the superpotential is a function of the D3-brane location, not the 7-brane location.) The potential for the coupled system of Kähler modulus and 3-brane coordinates is derived from W and the Kähler potential K :

$$K = -3 \log e^{4u} = -3 \log(\rho + \bar{\rho} - \gamma k(Y, \bar{Y})/3), \tag{3.4}$$

in the usual way,

$$V = e^K \left(g^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} - 3|W|^2 \right), \quad D_\alpha W = \partial_\alpha W + W \partial_\alpha K. \tag{3.5}$$

From now on, we will absorb the constant $\gamma \equiv \mu_3 \kappa_{10}^2$ into k .

Let us review how the dependence of (3.1) on the brane location was obtained by Baumann et al. [18], focusing on the 7-brane case for definiteness. The nonperturbative physics comes from strong coupling dynamics on a 7-brane, whose strength is controlled by the effective 4D gauge coupling after the wrapped 4-cycle Σ is integrated over:

$$\frac{1}{g^2} = \frac{\mu_3 V_\Sigma^w}{8\pi^2}. \tag{3.6}$$

Here V_Σ^w is the warped volume of Σ . The authors of [18] calculated how V_Σ^w is modified by the backreaction of the mobile D3-brane tension, and thus how the strength of W_{np} varies with the location of the D3.

The tree-level correction to the warped volume was determined straightforwardly in terms of the perturbation to the warp factor $h \equiv e^{-4A}$ by the brane's tension:

$$\delta V_\Sigma^w = \int_\Sigma d^4 Y \sqrt{g^{\text{ind}}} \delta h. \tag{3.7}$$

This *linear* dependence will be of key importance for us. The effective coupling constant on the 7-brane then enters the superpotential in the usual *exponential* form characteristic of instantonic corrections,

$$A(Y) = A_0 f^{1/n} \propto \exp(-\mu_3 V_\Sigma^w). \tag{3.8}$$

The dependence of the potential on the axion $b \equiv \text{Im } \rho$, in principle determined by the backreaction of C_4 , was then fixed via holomorphy.

This discussion so far has applied to D3-branes, which in the absence of moduli stabilization feel no force. In [19, 20] it was argued that the nonperturbative potential for

an *antibrane*, despite the absence of supersymmetry, takes the same form (3.5) with the function $f(Y)$ exchanged for its complex conjugate. This follows because the *tension* of the brane and antibrane are identical, and so the contribution to (3.7), which is determined by a gravitational perturbation, is unchanged at leading order.

Instead the sign of the C_4 fluctuation flips, changing the sign of the contribution that was fixed by holomorphy in the supersymmetric case. As was noted in [20], this affects only a largely insignificant part of the physics, namely the fixed value of the axion b ; other than knowing that it is fixed at some value, this will play no role for us.

As a result, the nonperturbative effects on a single $\overline{D3}$ -brane can be determined. We would like to push this a step further, and determine the effects on a stack of $\overline{D3}$ -branes as they expand into a 5-brane. As discussed following equation (2.23), the embiggened brane appears as an effective 3-brane with tension that does not match its charge due to the extra contribution to the tension from the 5-brane's extent over the two-sphere.

However, as we have just reviewed, the tension is far more important for calculating the nonperturbative physics than the charge. The former determines the coupled equations between the volume modulus and the brane coordinates, while the latter merely shifts the fixed value of the axion b , which is not important to our discussion. It is the tension we need to calculate the perturbation to the warped volume (3.7).

We shall thus focus entirely on the tension, disregarding the details of what happens to b . Since the δV_Σ^w depends linearly on this tension, we obtain for the warped volume of Σ the single-brane result multiplied by $N_3(\psi)$,

$$\delta V_\Sigma^w = N_3(\psi)(\delta V_\Sigma^w)_0. \tag{3.9}$$

This quantity, however, enters the nonperturbative physics in the *exponent*, leading to:

$$A(Y) = A_0 f(Y)^{N_3(Y)/n}. \tag{3.10}$$

This is the key result, and encodes how the nonperturbative potential is modified by the embiggened brane as its tension changes.

3.2 7-brane embedding

We shall find it useful to pick a particular 7-brane embedding so that we have a concrete value of the function f . Various 7-branes can break the $SO(4)$ isometry of the S^3 at the tip in various ways. Because the embiggened brane expands to fill a set of S^2 s foliating this S^3 , it is convenient to pick a 7-brane embedding that preserves the $SO(3) \subset SO(4)$ that acts on this S^2 . If one does otherwise, different points on the 5-brane will feel a different potential and it will deform; we choose to avoid this complication.

A natural choice is the so-called ‘‘simplest Kuperstein’’ embedding [31]:

$$f = z_1 - \mu, \tag{3.11}$$

which obviously preserves the $SO(3)$ acting on z_2, z_3 and z_4 . This embedding has vacua for both D3 and $\overline{D3}$ -branes at $z_1 = \pm\epsilon$ [19]. To make this embedding compatible with the

symmetry of the embiggened brane, we identify $z_1 = -\epsilon \cos \psi$; the brane decay will thus correspond to motion from one vacuum to another, $z = -\epsilon$ to $z = \epsilon$.

The parameter μ expresses how far down the throat the 7-brane reaches: $r_{\min} \sim |\mu|^{2/3}$. Strictly speaking μ is determined by the flux-mediated moduli stabilization that fixes the 7-branes, but as far as we are concerned it is essentially a free parameter. We can, however, characterize its possible values somewhat: in assuming that this 7-brane has the dominant effect over any other branes or dynamics in the compact geometry, we are assuming it penetrates at least some distance into the throat. An embedding with $|\mu| > 1$ corresponds to a 7-brane coming only a little ways down the throat, while $|\mu| < 1$ is a “deep throat” 7-brane. For the very small value $|\mu| < \epsilon$, the 7-brane actually passes through the tip [31]; we shall in general assume $|\mu| \gg \epsilon$ so the brane doesn’t go too deep in the throat, which we expect to be the generic situation.

3.3 Complex structure and Kähler parameters

There are a number of distinct parameters affecting the physics. The details of the values of these parameters are in general dependent on properties of the background outside the throat, including the various other three-form fluxes that stabilize the complex structure moduli, 7-brane moduli and dilaton. In our approximation the parameters are seven in number: the three-form fluxes in the throat M and K , the nonperturbative sector parameters W_0 and A_0 , the 7-brane location μ and the dilaton g_s (all four of which are fixed by unspecified fluxes) and the number of $\overline{D3}$ -branes p .

The two derived parameters that will be most important for us are the complex structure modulus ϵ , which is determined by K , M and g_s , and the Kähler modulus ρ , primarily determined by W_0 and A_0 , though as we shall review in the next section, the other parameters can have an effect as well. We shall mainly be focused on ϵ and ρ (as well as the flux superpotential W_0) so we review what might constitute “natural” values for these quantities.

For ρ , we have $\rho^{1/4} \sim e^u$ giving the overall length scale of the entire compactification R above the string scale $l_s \equiv \sqrt{\alpha'} = 1$ in our units. As we shall review shortly, for a standard KKLT compactification we have

$$a\rho = \log \left| \frac{A_0}{W_0} \right|. \tag{3.12}$$

To trust our geometric description of the model, we require $e^u \gg 1$; because of the logarithmic dependence, even relatively modest volumes can lead to A_0/W_0 being extremely large. Thus there is the well-known tension between the desire for large-volume solutions and the fine-tuning of the parameters involved.

The ratio between the scale of physics at the bottom of the warped throat and that at the top is determined by $e^{A_{\text{bott}}}/e^{A_{\text{top}}} \sim \epsilon^{2/3} e^u / \sqrt{g_s M}$ [45]. If we take the warped throat to correspond to a scale like the ratio of the electroweak and Planck scales, $M_{\text{EW}}/M_{\text{Pl}} \sim 10^{-16}$, we obtain

$$\epsilon \sim 10^{-24} (g_s M / \sqrt{\rho})^{3/4} \sim 10^{-22} - 10^{-26}, \tag{3.13}$$

for various reasonable values of $g_s M$ and ρ ; since according to the definition (2.5) $\log \epsilon \sim -\pi K/g_s M$, it follows that K must then be larger than $g_s M$ by a factor of 50 or so. As is well-known, a warped throat can generate such an exponential hierarchy with a quite reasonable set of values for the fluxes. Depending on the physics of the warped throat in question, ϵ closer to one may also be of interest and does not require significant fine tuning.

4 Structure of the complete potential

We now turn to our primary interest: the combination of the last two sections, which is the effect of the nonperturbative moduli stabilization on the antibrane decay process of KPV. We shall use the expression (3.10) in the potential (3.5), and compare the resulting potential to that for the brane/flux annihilation (2.23). We are interested in two degrees of freedom: the collective coordinate ψ standing for the transverse position of the embiggened brane, which will now take the place of the general brane coordinate Y , and the volume modulus ρ . The two pieces of the potential are,

$$V = V_{\text{bf}} + V_{\text{np}}, \tag{4.1}$$

where we can use the results from [20] to write the non-perturbative potential at the tip of the KS geometry as

$$V_{\text{np}} = \frac{a|W_0|^2 e^{-8u}}{|\omega|^2} \left(\frac{ae^{4u}}{3} + 2 + \omega + \bar{\omega} + G(\psi) \right). \tag{4.2}$$

Here we have defined the ratio between the flux superpotential and nonperturbative superpotential,

$$\omega \equiv \frac{W_0}{A(\psi)e^{-a\rho}}, \tag{4.3}$$

which plays a key role, as well as the real function of the brane coordinates,

$$G(\psi) \equiv \frac{1}{a} k^{a\bar{b}} \partial_a \zeta \partial_{\bar{b}} \bar{\zeta}, \tag{4.4}$$

where we have included the new term from the embiggening (3.10) in

$$\zeta \equiv -\log \left(\frac{A(\psi)}{A_0} \right) = -\frac{N_3(\psi)}{n} \log f(\psi), \tag{4.5}$$

and the geometric Kähler potential can be written as

$$k^{a\bar{b}} \equiv \frac{1}{Q} (\epsilon^2 \delta^{a\bar{b}} - z^a \bar{z}^{\bar{b}}), \quad Q \equiv \frac{2^{1/6}}{3^{1/3}} \mu_3 \kappa_4^2 \epsilon^{4/3}, \tag{4.6}$$

in terms of the conifold coordinates z^a (2.6), which are in general complex but become real at the tip of the geometry.

Meanwhile we note that V_{bf} (2.23) depends on the volume ρ as,

$$V_{\text{bf}} = \frac{\mathcal{C}_0}{(\rho + \bar{\rho})^2} (N_3(\psi) - Q_3(\psi)) \equiv \frac{D(\psi)}{(\rho + \bar{\rho})^2}. \tag{4.7}$$

As is well-known, an antibrane alone tends to decompactify the space. Before turning to the potential as a function of ψ , let us examine its behavior as a function of ρ .

4.1 Volume-dependence of V_{np}

As was shown in [20], the imaginary part of the $\partial_\rho V$ equations does nothing but require that the ratio of flux to nonperturbative superpotentials ω is real. This is realized by adjusting the axion $b \equiv \text{Im } \rho \sim \int_\Sigma C_4$ to cancel out the remaining phase; b appears nowhere else and so is always available to do this. From now on, we will ignore b , take ω to be real, and for simplicity use ρ simply to refer to what is properly its real part: $\rho = e^{4u}/2$.¹⁰

For the variation of the potential with respect to the volume, we find

$$\partial_\rho V = \frac{-a|W_0|^2}{24\rho^3\omega^2} (3G(2 + 2a\rho) + (4 + 2a\rho)(3 + 2a\rho + 3\omega)) - \frac{D}{4\rho^3}. \quad (4.8)$$

Equating this to zero, we obtain a quadratic equation for ω . We will be working consistently within the large-volume limit $\rho \gg 1$, where the solutions are

$$\omega \approx -\frac{a^2|W_0|^2\rho}{2D} \left(1 \pm \sqrt{1 - \frac{8}{3} \frac{D}{a|W_0|^2} \left(1 + \frac{3G}{2a\rho}\right)} \right). \quad (4.9)$$

This solution has a few important properties, the most immediately evident of which is that it has no solution if the parameters of the problem conspire to make the argument of the square root negative. As we shall see, this provides a key constraint on the stability of the decay process.

The nonperturbative potential (4.2) has two important limits, depending on whether G or ρ is larger. As we shall describe, when $\rho \gg G$, which we will call the volume-dominated case, the parameter ω typically has a magnitude of the same order as ρ , while for $G \gg \rho$, we call the nonperturbative potential brane-dominated and $|\omega|$ is instead of the same order as G .

Let us describe the function G in more detail. Using (4.4), (4.5) we find

$$G(\psi) = \frac{1}{aQ} \left(\epsilon^2 \delta^{a\bar{b}} - z^a \bar{z}^{\bar{b}} \right) \partial_a \zeta \partial_{\bar{b}} \bar{\zeta}, \quad (4.10)$$

with

$$\partial_a \zeta = \frac{z^a}{nz^1} \frac{\partial}{\partial z^1} (N_3 \log f), \quad (4.11)$$

and using $dz^1/d\psi = \epsilon \sin \psi$, the reality of the z^a at the tip and $z_a z_a = \epsilon^2 - |z^1|^2 = \epsilon^2 \sin^2 \psi$, we obtain

$$G = \frac{1}{an^2Q} \left(\log f N_3'(\psi) + \frac{\epsilon \sin \psi N_3(\psi)}{f} \right)^2. \quad (4.12)$$

The $N_3'(\psi)$ term was absent in the case of a fixed number of branes not undergoing an embiggening process.

Let us estimate the size of $G(\psi)$. At the beginning and end points of the brane decay we have $\sin(\psi = 0, \pi) = N_3'(\psi = 0, \pi) = 0$, and consequently $G = 0$ there. At a generic

¹⁰The real part of ρ also includes the geometric Kähler potential $k(Y, \bar{Y})$, but this is constant over the tip, and we can absorb this constant into a rescaling of A_0 .

value of ψ , both N_3 and N'_3 are of order M ; the latter can cross zero. Assuming $|\mu| \gg \epsilon$, these terms are approximately $(M \log \mu)/\epsilon$ and M/μ , and the former term (which is the new term) is generically larger. (Note that if N'_3 crosses zero, it will be smaller in the vicinity of that point.) If we take the larger term to dominate, we get

$$G \approx \frac{|\log \mu|^2 N'_3(\psi)^2}{an^2 Q}. \quad (4.13)$$

Thus estimating the order of magnitude of G , we have

$$G \sim \epsilon^{-4/3} M^2 \log^2 \mu. \quad (4.14)$$

4.2 Volume dominance and the endpoints of brane decay

We consider first the situation where $\rho \gg G$. This always occurs at least at two very important points: the beginning and end of the brane decay, $\psi = 0, \pi$ where $G = 0$ precisely.

The function ω then has the solution

$$\omega \approx -\frac{a^2 |W_0|^2 \rho}{2D} \left(1 - \sqrt{1 - \frac{8}{3} \frac{D}{a |W_0|^2}} \right), \quad (4.15)$$

where for reasons of matching the supersymmetric solution, as we describe momentarily, we have chosen the negative sign on the square root. It is readily apparent that real solutions do not exist for all values of the parameters; when solutions do exist, it is straightforward to verify that

$$\omega = -\frac{2}{3} a \chi \rho, \quad (4.16)$$

where $1 \leq \chi \leq 2$ depends on $D/|W_0|^2$. The nonperturbative potential then has the form

$$V_{\text{np}} = -\frac{3}{8} \frac{2\chi - 1}{\chi^2} \frac{|W_0|^2}{\rho^3}, \quad (4.17)$$

and is always negative.

4.2.1 Endpoint of brane/flux annihilation

At the endpoint of the brane/flux annihilation, the system is entirely supersymmetric, consisting of no antibranes, and branes only sitting in the minimum of the nonperturbative potential; moreover, the perturbative potential vanishes, giving $D(\psi = \pi) = 0$. We then see that, as claimed, the negative sign branch contains the proper smooth limit:

$$\omega_f = -\frac{2}{3} a \rho_f, \quad (4.18)$$

so $\chi_f = 1$. Recalling the definition of ω , this reduces to

$$\left| \frac{W_0}{A(\psi = \pi)} \right| e^{a\rho_f} = \frac{2}{3} a \rho_f, \quad (4.19)$$

the standard KKLT-type relation for the volume modulus. As usual, a large-volume solution is not generic and can only be obtained by a tuning of the parameters. Assuming a large volume and taking the logarithm of both sides gives

$$a\rho_f = \log \left| \frac{A_0}{W_0} \right| + \frac{M-p}{n} \log |\mu| + \dots, \quad (4.20)$$

where we recalled that $A(\psi) = A_0 f(\psi)^{N_3(\psi)/n}$ and $N_3(\psi = \pi) = M - p$, and took $\mu \gg \epsilon$ so that $f(\psi) \approx \mu$; we have neglected terms of order $\log \rho$.

In a traditional KKLT-type stabilization, it is the smallness of W_0 that drives the large size of ρ ; this can also be achieved with large A_0 or a combination of the two. As we shall see shortly, the magnitude of W_0 (but not A_0) is constrained by the dynamics. Note additionally that the D3-brane tension also contributes to the stabilized volume; indeed for natural values of the parameters where A_0 and W_0 are not far from unity, it can have a significant effect.

4.2.2 Initial point of brane/flux annihilation

For the point at the beginning of the process where the $\overline{D3}$ -branes have not yet started to decay, we still have $G = 0$ but $D > 0$ in general. The problem reduces to the standard coupling of the nonperturbative moduli stabilization to an uplifting set of branes explored in KKLT [6].

Most significantly, we readily see from equation (4.9) that there is an upper limit on D to permit such a solution:

$$\frac{3}{8}a|W_0|^2 \geq D(\psi = 0), \quad (4.21)$$

corresponding to the well-known fact that if the antibrane tension is too strong, it will overwhelm the moduli stabilization and drive spontaneous decompactification [6]. Thus equation (4.21) corresponds to the *decompactification constraint* on the initial vacuum. Up to numerical factors of order unity, the decompactification constraint for the initial vacuum is

$$|W_0|^2 \geq \frac{p}{M} \frac{\epsilon^{8/3}}{g_s^3 M}. \quad (4.22)$$

A large number of antibranes tends to decompactify the space, but this is strongly mitigated by a small complex structure parameter ϵ reducing their effective tension.

We will assume this bound is satisfied so that such a vacuum exists. We then find

$$\omega_i = -\frac{2}{3}a\chi_i\rho_i, \quad (4.23)$$

where as before χ_i depends on $D_i/a|W_0|^2$ and for permitted values, varies merely between $1 \leq \chi \leq 2$. We thus have for the initial volume,

$$a\rho_i = \log \left| \frac{A_0}{W_0} \right| + \frac{p}{n} \log |\mu| + \dots, \quad (4.24)$$

where we dropped $\log \chi$ as being insignificant.

Notice that if we assume that the large volume is primarily due to W_0 being very small as $\rho \sim \log |1/W_0|$, we can easily run afoul of the decompactification constraint. The ratio A_0/W_0 , however, is unfixed and so in principle can be tuned to produce any volume despite the constraint; a large A_0 is in general required.

Also note that the *difference* between final and initial volumes is

$$\Delta\rho \approx \frac{M - 2p}{2\pi} \log |\mu|. \tag{4.25}$$

This is a key result: in order for the volume not to change substantially during the brane/flux annihilation process, we must have

$$\rho \gg \left| \frac{(M - 2p)}{2\pi} \log |\mu| \right|. \tag{4.26}$$

Since M may be of order hundreds or more, as we require $g_s M \gg 1$ with g_s small, this volume change becomes quite relevant for values of ρ that are not too large. This is our second key result: a brane/flux decay with only modestly large volume will in general change its volume during the course of the decay process.

Before moving on to a study of the middle of the decay, let us consider the initial cosmological constant in the system. This comes from an interplay between V_{bf} , which is positive, and V_{np} , which is negative:

$$\Lambda = V_{bf,i} + V_{np,i} = |V_{np,i}|(\lambda - 1), \tag{4.27}$$

where we have defined

$$\lambda \equiv |V_{bf,i}|/|V_{np,i}| \sim \frac{D_i \rho_i}{|W_0|^2} \sim \frac{p}{M} \frac{\epsilon^{8/3}}{g_s^3 M} \frac{\rho_i}{|W_0|^2}. \tag{4.28}$$

Evidently, $\lambda > 1$ leads to an initial metastable state with a positive cosmological constant, while $\lambda < 1$ implies a negative cosmological constant. A realistic present-day vacuum energy, in which case the brane annihilation process would correspond to the decay of our world, would require λ extremely close to one,

$$|W_0|^2 \sim D_i \rho_i, \tag{4.29}$$

which, it is nice to verify, satisfies the decompactification constraint (4.21) with a factor of ρ to spare. This constraint prevents existence of metastable vacua with larger positive cosmological constant, characterized by $\lambda \sim \rho$ or greater.

4.2.3 General volume dominance

During the brane decay $0 < \psi < \pi$, it is possible for G to grow and overwhelm the volume, changing the form of the potential. Given the estimate for G (4.14), this will not happen and the volume will continue to dominate the potential for

$$\rho \gg \epsilon^{-4/3} M^2 \log^2 \mu. \tag{4.30}$$

For the modulus ϵ chosen to solve the hierarchy problem, we expect the right-hand-side to be in excess of 10^{32} . Thus volume-domination of the nonperturbative potential persists only if the volume is very large indeed; we need length scales 10^8 times the string scale for universal volume-domination, bringing us into the realm of large extra dimension models. (The fine-tuning of the parameters W_0 and A_0 required to obtain such a large volume is truly extraordinary, as well.) We shall therefore largely focus on the brane-dominated nonperturbative potential.

4.3 Brane dominance

Away from the beginning and the end of the decay, we will have $G \gg \rho$ if

$$\epsilon^{-4/3} M^2 \log^2 \mu \gg \rho, \tag{4.31}$$

and as just argued we expect this situation to be the easier to realize. The solution (4.9) for ω becomes

$$\omega \approx -\frac{a^2 |W_0|^2 \rho}{4D} \left(1 - \sqrt{1 - \frac{4DG}{a^2 |W_0|^2 \rho}} \right), \tag{4.32}$$

which takes the value

$$\omega = -\chi' G, \tag{4.33}$$

with $1 \leq \chi' \leq 2$ when a solution is possible. The nonperturbative potential in this limit takes the form

$$V_{\text{np}} \approx -\frac{a(2\chi'(\psi) - 1) |W_0|^2}{4\chi'(\psi)^2 \rho^2 G(\psi)}. \tag{4.34}$$

In the next section, we shall describe the consequences of the nonperturbative potential on the decay of the embiggened antibranes.

4.4 Validity of the nonperturbative expansion

The nonperturbative term going like $e^{-a\rho}$ in (3.1) is in principle only the leading term in an expansion of exponentials in ρ ; it is only valid to neglect other terms when the leading term is small. At large volume $e^{-a\rho}$ is always small, but one may wonder whether $A(\psi)e^{-a\rho}$ might still be large, thus invalidating the expansion.

Using the definition (4.3) of ω , the size of the nonperturbative term is

$$A(\psi)e^{-a\rho} = \frac{W_0}{\omega}, \tag{4.35}$$

and this furnishes an upper bound on $|W_0|$ to be less than ω , which we have now shown goes like the larger of ρ and G . Since $|W_0|$ is generally taken small to support large volume solutions, this is not in general a difficult constraint to satisfy.

5 Moduli-stabilized brane/flux transitions

In what follows, we will study the ψ -dependence of the total potential with (4.9) enforcing that it sits at an extremum in the ρ -direction (if such an extremum exists). Thus we are assuming that the $\partial_\rho V = 0$ equation, which held at $\psi = 0$ to define the initial vacuum, remains satisfied and adjusts the system accordingly as the the brane rolls down a slope in ψ . We expect that an analysis involving both ρ and ψ as dynamical fields, beyond the scope of this paper, would reflect the same phenomena we uncover here.

As we have already seen, several interesting things can occur during the decay of the embiggened brane under the influence of the nonperturbative potential. We have reviewed how there is a limit on the initial (positive) cosmological constant to have a metastable vacuum at all; in the next subsection, we shall describe how even if this constraint is satisfied, certain positive cosmological constant models violate the related bound during the decay, and thus decompactify. We shall then discuss how for cosmological constants near zero it is possible for the nonperturbative potential to stabilize an otherwise perturbatively unstable solution, potentially saving a supersymmetry-breaking universe like our own from quick decay. Total dominance of the nonperturbative potential, however, occurs only for negative initial cosmological constants. Finally, we note how even if the transition completes, the overall volume can be changed. In the next section, we present an example illustrating these possibilities.

5.1 Spontaneous decompactification

For the cases where $G \gg \rho$ during the decay (or even $G \sim \rho$), we can run into trouble satisfying the $\partial_\rho V = 0$ equation (4.9). We assumed that $|W_0|$ took a value such that a solution existed before the decay, requiring (4.21); even so, we can run into difficulties in the middle of the decay process.

The decompactification constraint for $G > \rho$ is modified: solutions to (4.32) require

$$a^2 |W_0|^2 \rho > 4D(\psi)G(\psi), \tag{5.1}$$

and using the estimate

$$D \approx \frac{\epsilon^{8/3}}{g_s^3 M}, \tag{5.2}$$

we can parameterize the constraint as approximately,

$$|W_0|^2 > \frac{\epsilon^{4/3} M \log^2 \mu}{g_s^3 \rho}. \tag{5.3}$$

It is entirely possible for $|W_0|$ to be such that the initial metastable vacuum for the antibranes exists, but the constraint is then violated during the course of the decay (for an example, see figure 5 in the next section). We can parameterize this in terms of the initial cosmological constant parameter λ ; we find

$$\lambda < \frac{p}{M} \frac{\rho^2}{G}, \tag{5.4}$$

while a metastable vacuum at the outset only required $\lambda < \rho$.

The inability to satisfy the equation (4.9) at some point during the decay means the potential in the ρ -direction no longer has a local minimum; the potential has “opened up” to a slope leading down towards large volume. Although in principle one would need to solve the full dynamical coupled system of ψ and ρ including kinetic terms to obtain the full motion, we generically expect that once the system finds itself without a minimum in the ρ -direction, it will roll to large volume.

Thus it is possible for the brane/flux annihilation process to spontaneously decompactify the otherwise metastable system. The decay of the embiggened brane presumably completes, but the system is no longer of phenomenological interest; it has decayed to ten-dimensional flat space. Metastable vacua with large initial cosmological constant are the most in danger of this.

Can this occur for an initial state with vacuum energy near zero? This coincides with $\lambda = 1$, implying for stability

$$\epsilon^{-4/3} \frac{M}{p} M^2 \log^2 \mu < \rho^2, \tag{5.5}$$

where we used (4.14) to estimate G .

This requires a substantial volume. For example, for the case of the throat solving the hierarchy problem, we need something of the order $\rho > 10^{16}$, or length scale 10^4 times the string scale. While not as large a volume as was necessary for the nonperturbative potential to become volume-dominated, this is still a substantial volume representing a tremendous tuning of A_0 relative to W_0 .

Thus we see that a set-up with a realistic cosmological constant may spontaneously decompactify before the brane decay can complete for a modest compact volume. In general, a deep throat encourages spontaneous decompactification, while a large volume discourages it; the former is much more generic in the space of parameters.

5.2 Nonperturbative dominance

Assuming the system satisfies the decompactification constraint throughout its evolution, the next question to ask is whether the nonperturbative potential is smaller than, similar to or much larger than the perturbative potential. In the more generic case of $G \gg \rho$, we find

$$\frac{|V_{\text{np}}|}{|V_{\text{bf}}|} \sim \frac{|W_0|^2}{GD}, \tag{5.6}$$

$$\sim \frac{1}{\rho} \frac{|W_0|^2}{|W_0^{\text{min}}|^2}, \tag{5.7}$$

where $|W_0^{\text{min}}|^2$ is defined by the minimum allowed value for the decompactification constraint (5.3).

We readily see that if W_0 is near the smallest value allowed to prevent spontaneous decompactification, the nonperturbative potential is suppressed relative to the brane/flux potential by a factor of the large volume. In this region, then, the brane decay potential is unmodified from the original study of [1], and the decay goes through as described

there. Thus, it is entirely possible for brane/flux transitions to proceed unmodified by the nonperturbative physics.

In terms of the initial cosmological constant parameter λ , we have

$$\frac{|V_{\text{np}}|}{|V_{\text{bf}}|} \sim \frac{p}{M} \frac{\rho}{G} \frac{1}{\lambda}, \tag{5.8}$$

demonstrating that the $\lambda \gg 1$ systems with large cosmological constant, the ones in danger of decompactifying, are always dominated by the brane/flux potential during their evolution. Thus positive cosmological constant metastable states, if they do not decompactify, tend to decay in the original way described by KPV.

The vacua where V_{np} clearly dominates have $\lambda \ll 1$, that is, they have negative cosmological constant. These are less interesting phenomenologically, since our universe has positive vacuum energy and any hypothetical states in the early universe that decayed to our vacuum would have had larger energy still. V_{np} for $G \gg \rho$ has its ψ -dependence primarily from $-1/\rho^2 G(\psi)$, except at the endpoints, where it becomes $-1/\rho^3$. So, V_{np} is most negative near $\psi = 0$ and $\psi = \pi$, and has a less negative “bump” in the middle, of size ρ/G closer to zero; thus V_{np} tends to rise in the middle of the annihilation process, shutting down the decay and stabilizing the system at small ψ if it is dominant (see for example figure 2 in section 6).

It is, however, possible for models near $\lambda = 1$, a realistic cosmological constant, to have the two potentials similar sizes, $|V_{\text{np}}| \sim |V_{\text{bf}}|$. Even if the brane/flux potential would lead to unobstructed decay, the “bump” from the nonperturbative potential can in this case provide a stabilizing barrier. Thus it is possible for a universe with vacuum energy similar to ours which was perturbatively unstable from V_{bf} alone, to be stabilized perturbatively by the nonperturbative potential, as in figures 3 and 4.

Inflation in the context of the brane/flux annihilation decay was studied in [10] (see also [9]), and when the brane/flux potential dominates the story from there will be unchanged. Inflation obviously will not occur for nonperturbative-dominant cases, which have negative vacuum energy; it is possible that the regime where both potentials contribute could lead to an enhanced flat direction if they are tuned just right, but the caveats of the difficulty of such a tuning outlined in [10] still remain.

5.3 Finite volume change

While the brane decay is proceeding, the value in the ρ -direction that minimizes the potential may vary. The analogous equation to (4.20) and (4.24) for any point along ψ is

$$a\rho(\psi) = \log \left| \frac{A_0}{W_0} \right| + \frac{N_3(\psi)}{n} \log |\mu| + \dots, \tag{5.9}$$

where \dots contains $\log \rho$ or $\log G$, depending on which is larger. The corresponding volume change associated to the second term is of the order,

$$\Delta\rho \sim M \log \mu. \tag{5.10}$$

It is useful to note that G is of the order (4.14),

$$G \sim \epsilon^{-4/3}(\Delta\rho)^2, \tag{5.11}$$

which implies that in the volume-dominated case of $\rho \gg G$,

$$\frac{\Delta\rho}{\rho} \ll \frac{\epsilon^{2/3}}{\rho^{1/2}}, \tag{5.12}$$

and no significant volume change occurs. However, in the more generic case of brane-dominance, it is possible for such a change to occur. If $\log |A_0/W_0|$ is only of order hundreds or thousands, it will generically be the case that the volume change will be significant, even if $\log |\mu|$ is of order one.

Note that the *sign* of the volume change is determined by $\log |\mu|$: a 7-brane only going a little way into the throat will have $\log |\mu| > 0$ and hence $\Delta\rho > 0$, while a 7-brane penetrating far into the throat will produce $\Delta\rho < 0$. Since we expect the former to be more common, volume increase is more generic, but either is possible as in figure 6.

We also note the possibility that the volume may be dominated by the flux at all points: $N_3(\psi) \log |\mu| \gg \log |A_0/W_0|$ for all ψ . The volume would then simply be

$$a\rho(\psi) \approx \frac{N_3(\psi)}{n} \log |\mu|, \tag{5.13}$$

and would grow by a factor of M/p during the course of the decay. ($\log |\mu| < 0$ in this case would lead to negative volume, outside the bounds of our large-volume approximation.) Although this is a different scenario than the one usually envisioned in a KKLT-type model, it may actually be somewhat generic, as large $M \log \mu$ is substantially more natural than large $\log |A_0/W_0|$ due to the logarithm. However, we note that in this case $G \sim \epsilon^{-4/3} \rho^2$, implying $G \gg \rho^2$; one can then show that satisfying the decompactification constraint (5.1) implies that $\lambda < 1$ in (4.28), leading to an unphysical negative cosmological constant at the beginning and end of the transition.

6 Summary and example

In summary, we have found the following broad classes of dynamics for the moduli-stabilized embiggened brane system:

- Metastable vacua with large positive cosmological constant can spontaneously decompactify during the decay, returning to ten-dimensional flat space. If they do not decompactify, the brane/flux annihilation proceeds as originally described.
- Vacua with vacuum energy near zero can be unstable to decompactification, but can also be stabilized against perturbative decay by the nonperturbative physics.
- Vacua with negative cosmological constant are dominated by the nonperturbative potential, and the decay is shut down.

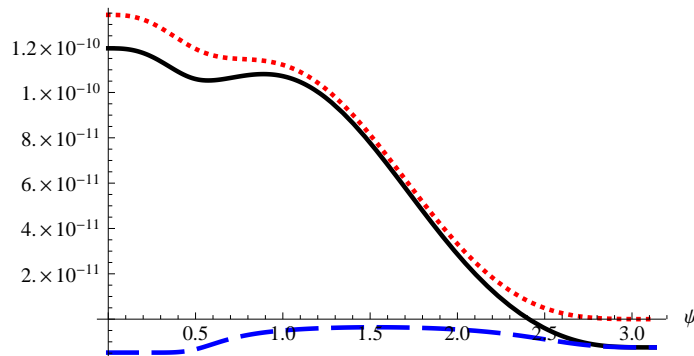


Figure 1. The full potential (black, solid) and its components V_{np} (blue, dashed) and V_{bf} (red, dotted) for $|W_0| = 0.2$, dominated by the brane/flux potential.

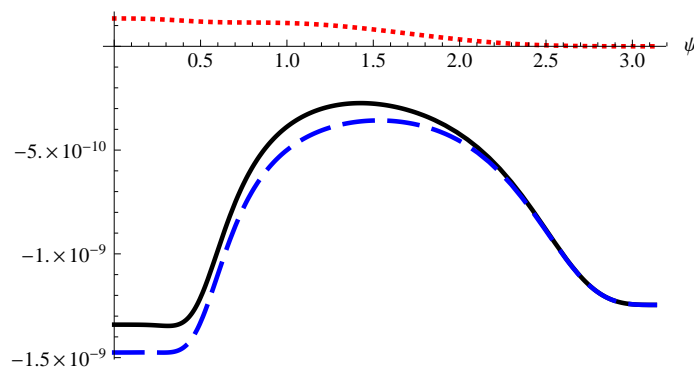


Figure 2. The full potential (black, solid) and its components V_{np} (blue, dashed) and V_{bf} (red, dotted) for $|W_0| = 2$, dominated by the nonperturbative potential.

- If the initial volume is only modestly large, it can grow or shrink during the course of a successful brane decay, with the sign of the change determined by how deep the 7-brane hosting nonperturbative physics penetrates into the throat.

For an example, consider the following choices of parameters:

$$\epsilon = 0.05, \quad \log \left| \frac{A_0}{W_0} \right| = 1000, \quad g_s = \frac{1}{10}, \quad M = 100, \quad \mu = 2, \quad \frac{p}{M} = 0.08, \quad (6.1)$$

where we will allow W_0 to vary in the vicinity of unity; the nonperturbative term (4.35) is always small. Note that this is not a particularly realistic model for solving the hierarchy problem, as the throat is extremely shallow; we chose it for computational convenience rather than realism.

For $|W_0| < 0.2$, the brane/flux potential is dominant; this is plotted in figure 1. Meanwhile for $|W_0| > 2$, the total potential is dominated by the nonperturbative contribution, as plotted in figure 2. In the former case the initial vacuum energy is positive, and the decay process proceeds unobstructed, while in the latter case the vacuum energy is negative and the decay is obstructed perturbatively. We choose to plot values where the smaller

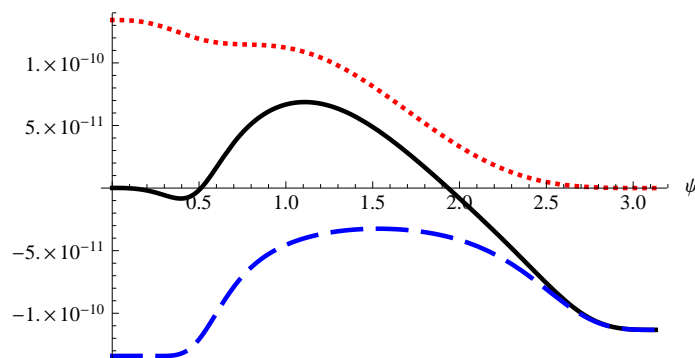


Figure 3. The full potential (black, solid) and its components V_{np} (blue, dashed) and V_{bf} (red, dotted) for $|W_0| = 0.603$, with initial vacuum energy close to zero.

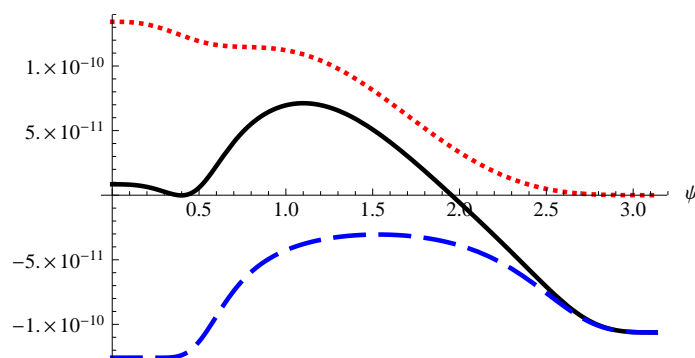


Figure 4. The full potential (black, solid) and its components V_{np} (blue, dashed) and V_{bf} (red, dotted) for $|W_0| = 0.584$, with metastable vacuum energy close to zero.

potential still makes some contribution for visual interest; as W_0 moves further away from the cross-over region, one potential or the other dominates completely.

For intermediate values both potentials contribute; in particular, in the vicinity of $|W_0| = 0.60325$ the initial cosmological constant is tiny as resulting from a near-perfect cancelation of the two contributions to the potential; the system embiggens only slightly to a metastable vacuum with a negative cosmological constant, figure 3. Perhaps of more interest is $|W_0| = 0.5837$ (figure 4), where the brane grows slightly and drops into a metastable vacuum with cosmological constant near zero.

One can also see that for $|W_0| < 0.025$, the argument of the square root in the solution for ω goes negative and the decompactification constraint is not satisfied (figure 5). Thus we see clearly: very small $|W_0|$ leads to decompactification, larger to perturbative-dominance with allowed decay, larger still to a mixture with vacuum energy near zero and largest of all to nonperturbative dominance with a negative vacuum energy and decay shut down.

Finally, we note that since ρ is of order 10^4 and $\Delta\rho = M \log \mu$ of order 10^2 , the volume changes on the 10^{-2} level during the decay, going from $\rho_i = 1005$ to $\rho_f = 1064$. We plot the change for $\mu = 2$ as well as $\mu = 1/2$ in figure 6, showing both a growth and shrinkage

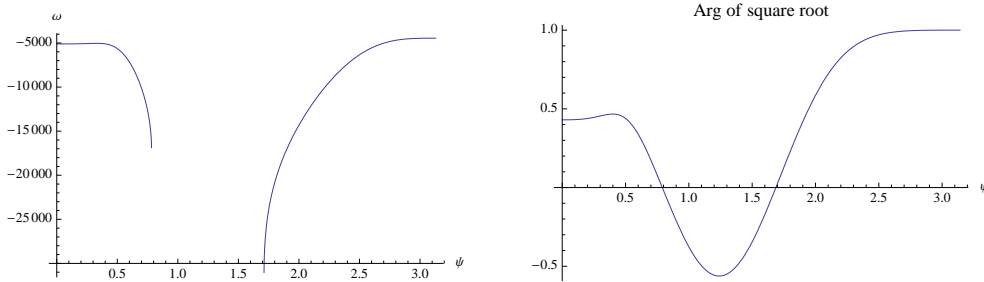


Figure 5. The function ω and the argument of the square root in its formula (4.9) for $|W_0| = 0.02$; note the gap where ω has no real solution, leading to spontaneous decompactification.

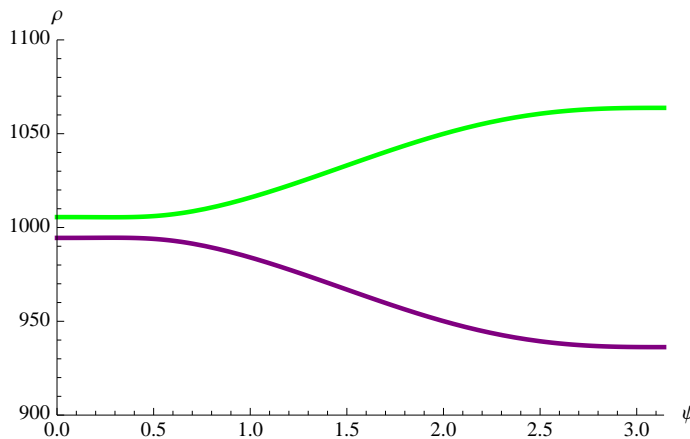


Figure 6. The variation in the Kähler modulus ρ for $\mu = 2$ (upper) and $\mu = 1/2$ (lower). Values further from unity lead to greater variation as $\log |\mu|$.

of the compact volume; values of μ farther away from unity will increase the effect.

7 Conclusions

We have demonstrated that the interplay of nonperturbative moduli stabilization and brane/flux annihilation can have a rich variety of effects. Our results have a number of consequences for the stability and decay of flux compactifications of string theory.

Stabilization of stacks of antibranes at positive or very near zero cosmological constant, which without nonperturbative physics may decay classically, has obvious phenomenological utility for construction of realistic string vacua. The existence of spontaneous decompactification, on the other hand, implies that any vacuum transitions in the early universe mediated by a brane/flux annihilation process are less stable than would otherwise be expected; this places more stringent bounds on the path such a transitioning universe could have taken to avoid ending up in ten dimensions at some point in its history. Moreover, a background that makes many such transitions will have changed its volume with each decay. If the nonperturbative physics is hosted not far down the throat in each case, this can

accumulate and cause the volume to grow to a size substantially above the string scale with less than the usual fine tuning required, although if this contribution dominates, another source of positive vacuum energy must be added.

These calculations could be expanded upon in a number of ways. We have performed the entire analysis assuming the existence of only one Kähler modulus; a generic compactification will have many, and it is possible that the interaction between them in addition to their effects on the brane decay could lead to novel effects. We have not studied the dynamical system including kinetic terms for the Kähler modulus as well as the brane coordinate; such a calculation could shed light, for example, on the process of spontaneous decompactification and whether it might be avoided in certain circumstances, for example if the decay occurs sufficiently quickly. We have not included the perturbative backreaction of the rest of the geometry outside the throat; this is one more possible source of forces on the brane system, as well. We anticipate, however, that the broad classes of phenomena we have outlined will persist in these more general cases.

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